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SOLUTION OF THE SIX ORIGINAL PROBLEMS PUBLISHED IN NO. SIX, VOL. III.

BY THE PROPOSER.

SOLUTION OF I. Let d, d' be the sun's distance from any two circular planetary orbits; v, v', the orbital velocities of the two planets; m, m', their masses; g, g', their intensities of gravity towards the sun; let i, i' be those parts of gravity which are converted into the orbital forces; s, the orbital distance over which a planet meves, while gravity is being transmitted from the sun to the planet; and let F, F' be the required orbital forces and R, R', the required resisting forces.

With the distances d and s form a right-angled parallelogram, the diagonal of which will represent the compound force of gravity; this force may be resolved into two constituent forces; one called the central force; the other, the orbital force: the central force varies as the intensity of gravity; the orbital force varies as its intensity multiplied into the aberrating velocity. Also in the planetary system, the diagonal and d are so very nearly equal, that the latter can be used for the forme without any appreciable error.

When the aberrating velocity is constant, the orbital force must vary as the force of gravity; therefore

$i : i' :: g : g'$.
Newton's law gives $g : g' :: m \div d^2 : m' \div d'^2$;
hence $i : i' :: m \div d^2 : m' \div d'^2$. . . . . (1)

But in circular orbits, the aberrating velocities are equal to the planetary velocities; also by the well known law of planetary velocity, we have

$v : v' :: 1 \div \sqrt{d} : 1 \div \sqrt{d'}$;
multiply by (1) $vi : v'i' :: m \div \sqrt{d^5} : m' \div \sqrt{d'^5}$.
But $F : F' :: vi : v'i'$;
therefore $F : F' :: m \div \sqrt{d^5} : m' \div \sqrt{d'^5} :: R : R'$.

In the problem, it is assumed that the permanent form of the orbits is circular; hence the resistances in any two circular orbits must be, not only proportional, but exactly equal to the propelling orbital forces; hence $R = F$ and $R' = F'$.

SOLUTION OF II. Let D, D' be the densities of the ethereal medium, at the respective distances of any two planets from the sun.

As the resistance depends upon the mass, square of the velocity, and density, we have

$R : R' :: mv^2D : m'v'^2D'.$

But $v^2 : v'^2 :: 1\div d : 1\div d'$;

hence $mv^2D : m'v'^2D' :: mD\div d : m'D'\div d'$;

therefore $R : R' :: mD\div d : m'D'\div d'$.

By I. we have $R : R' :: m\div\sqrt{d^5} : m'\div\sqrt{d'^5}$;

therefore $D : D' :: 1\div\sqrt{d^3} : 1\div\sqrt{d'^3}$.

Therefore the density of the ethereal medium must vary inversely as the cube of the square root of the suns' distance, in order to render the two antagonistic forces in circular orbits, equal.

Solution of iii. First, find the intensity of the earth's gravity towards the sun, compared with the intensity of gravity towards the earth's center, which may be assumed equal to unity. We have

$$\frac{91430000}{3955.94943182} = 23364.80826992676 = d.$$

We also have $\frac{\text{sun's mass}}{d^2} = \frac{314760}{d^2} = g$ $= .00057657405181 9522$; that is, the gravity towards the earth's center is to the gravity of the earth towards the sun as $1 : G$.

Second, find the intensity of the accelerating orbital force, exerted upon the earth. The earth moves in its orbit 8976.11926 miles, while gravity is transmitted over the distance d; hence we have

$$8976.11926 : 91430000 :: 1 : 10185.91635788,$$

or

$$\frac{\text{orbital force}}{\text{central force}} = \frac{1}{10185.91635788};$$

that is, the earth is accelerated in its orbit, by the $\frac{1}{10185.91635788}$th part of its central or gravitating force toward the sun.

This intensity, by the following proportion, can be expressed in terms of terrestrial gravity.

$10185.91635788 : 1 :: .000576574051819522$
$: .00000005660502517021 7 = G_1.$

G_1 is the intensity of the earth's accelerating orbital force. (the intensity of gravity towards the eareh's center being unity.)

Third, find the weight of the whole earth $=$ 13, 502, 160, 021, 599, 966, 659, 335, 933 ℔. This multiplied by G_1 gives

764, 290, 107, 874, 962, 825 ℔ $= p$.

This is the pressure in pounds weight in the direction of the earths orbital motion; and is also the resistance in pounds weight necessary to maintain the earth in a circular orbit.

Solution of iv. In an elliptic orbit, the angular velocity of a body around the lower focus, varies inversely as the square of the distance from the focus. (See Dr. Stewart's First Math. Tract, Prop. VI.)

The real velocity in an ellipse may be resolved into two velocities; one in the direction of the radius vector, and the other in a direction perpendicular to the radius vector; the former has no aberrating effect; the latter gives rise to the aberrating velocity of gravity.

The aberrating velocity varies inversely as the distance of the moving body from the focal force.

Let d and d' be the distances from the focal force of any two points in an ellipse; let V, V' be the angular velocities of a body at those two points, and let v and v' be the aberrating volocities, or those parts of the velocities which are at right angles to the radius vector.

The actual velocities at right angles to the radius vector must be as the angular velocities multiplied into the respective distances: hence

$$v : v' :: Vd : V'd'.$$

But $V : V' :: 1 \div d^2 : 1 \div d'^2$;

hence $Vd : V'd' :: 1 \div d : 1 \div d'$;

therefore $v : v' :: 1 \div d : 1 \div d'$. . . . . (1)

But the intensity of the aberrating force varies inversely as the square of the distance; hence $i : i' :: 1 \div d^2 : 1 \div d'^2$;

multiply by (1) and we have

$$iv : i'v' :: 1 \div d^3 : 1 \div d'^3;$$

by prob. I. $F : F' :: iv : i'v'$;

therefore $E : F' :: 1 \div d^3 : 1 \div d'^3$.

Thus it is proved that the aberrating or orbital force, in an elliptic orbit, varies inversely as the cube of the distance from the focal force.

Solution of v. As the square of the velocity enters as a function in determining the resistance, it will be necessary to show how this function is obtained.

Let d, d' be any two distances of a body from the lower focus, k, k', the distances from the upper focus; h, h', the distances of the apsidal points from the lower focus; p, p, the perpendiculars let fall from the lower focus on the two tangents drawn from the points in the orbit at the distances d and d', and v and v', the elliptic velocities of the moving body at the two points.

From the first Math. Tract of Dr. Matthew Stewart, Prop. 21, Cor., we

have $p^2 : h \times h' :: d : k$;

hence $p^2 = (h \times h' \times d) \div k$.

Also $p'^2 : h \times h' :: d' : k'$;

hence $p'^2 = (h \times h' \times d') \div k'$.

Therefore $p^2 : p'^2 :: \dfrac{h \times h' \times d}{k} : \dfrac{h \times h' \times d'}{k'} :: \dfrac{d}{k} : \dfrac{d'}{k'}$;

hence $\frac{1}{p^2} : \frac{1}{p'^2} :: \frac{k}{d} : \frac{k'}{d'}$;

but (see James Adams' Ellipse, centripetal forces, Prop. I., Cor. I.)

$$v^2 : v'^2 :: 1 \div p^2 : 1 \div p'^2;$$

therefore $v^2 : v'^2 :: k \div d : k' \div d'$. . . . . (1)

In determining the resistance, the density must be multiplied into the square of the velocity.

Let D, D' be the densities of the resisting medium, at the two points in the ellipse, and let R and R' be the respective resistances at those points.

By II. we have $D : D' :: 1 \div \sqrt{d^3} : 1 \div \sqrt{d'^3}$;
multiply by (1) $v^2 D : v'^2 D' :: k \div \sqrt{d^5} : k' \div \sqrt{d'^5}$.
But $R : R' :: v^2 D : v'^2 D'$;
therefore $R : R' :: k \div \sqrt{d^5} : k' \div \sqrt{d'^5}$.

In problems IV. and V., the mass is not taken into the calculation, because the mass in any one elliptic orbit, is considered constant, and therefore, cannot affect either of the variable forces.

SOLUTION OF VI. Let v', V, g', r' be respectively, the real velocity, the aberrating velocity, the intensity of the orbital force of gravity and the resistance, at the mean distance of a body in any elliptic orbit around the sun: let m = the mass of the earth and m' = the mass of the planet or comet revolving in the ellipse; let V', g'', and r'' be the aberrating velocity, the intensity of the orbital force of gravity, and the resistance, at the point in the ellipse where the two opposite forces are equal, and let x = the required length of the radius vector to the point of equal forces.

v' can be found from the proportion

$$1 \div \sqrt{a} : 1 \div \sqrt{a'} :: v : v'; \text{ hence } v' = v\sqrt{a} \div \sqrt{a'}.$$

In an elliptic orbit, at the mean distance, the aberrating velocity V = the angular velocity; hence we have

$$a' : b' :: v' : V :: v\sqrt{a} \div \sqrt{a'} : V; \text{ hence } V = b'v\sqrt{a} \div \sqrt{a'^3}.$$

By Newton's law of gravity, we have

$$m \div a^2 : m' \div a'^2 :: g : g'; \text{ hence } g' = m'ga^2 \div ma'^2.$$

Therefore $Vg' = b'vgm'\sqrt{a^5} \div m\sqrt{a'^7}$.

But, by problem IV., it is proved that

$$Vg' : V'g'' :: \frac{1}{a'^3} : \frac{1}{x^3};$$

hence $\frac{b'vgm'\sqrt{a^5}}{m\sqrt{a'^7}} : V'g'' :: \frac{1}{a'^3} : \frac{1}{x^3}$;

therefore $V'g'' = \frac{b'vgm'a'^3\sqrt{a^5}}{m\sqrt{a'^7}.x^3} = \frac{b'vgm'\sqrt{a^5}}{m\sqrt{a'}.x^3}$

equal the accelerating force at the distance x.

We also have $m \div \sqrt{a^5} : m' \div \sqrt{a'^5} :: r : r'$; $\therefore$ $r' = rm'\sqrt{a^5} \div m\sqrt{a'^5}$. And by problem V., we have

$$r' : r'' :: \frac{k}{\sqrt{a'^5}} : \frac{k'}{\sqrt{x^5}} :: \frac{a'}{\sqrt{a'^5}} : \frac{2a' - x}{\sqrt{x^5}};$$

or

$$\frac{rm'\sqrt{a^5}}{m\sqrt{a'^5}} : r'' :: \frac{1}{\sqrt{a'^3}} : \frac{2a' - x}{\sqrt{x^5}};$$

therefore

$$r'' = \frac{(2a' - x)rm'\sqrt{a^5}}{a'm\sqrt{x^5}}$$

= the resisting force at the distance x.

But at the distance x, the accelerating and resisting forces are (by hypothesis) equal. Therefore we have

$$\frac{b'vgm'\sqrt{a^5}}{m\sqrt{a'}.x^3} = \frac{(2a' - x)rm'\sqrt{a^5}}{a'm\sqrt{x^5}}.$$

This reduced becomes

$$x^3 - 4a'x^2 + 4a'^2x - v^2g^2b'^2a' \div r^2 = 0.$$

This equation, numerically solved, gives one of the values of x equal to the required radius vector. When the two forces are assumed equal in circular orbits, x never becomes less than the mean distance, nor greater than the aphelion distance. It is evident that there are two *radii vectores* of the same value, where the moving body is acted upon by two equal and opposite forces; from these two points, by the way of the aphelion, the aberrating force preponderates; and by the way of the perihelion, the resisting force is in excess. The sum of the resisting forces, in an elliptic orbit, is always greater than the sum of the aberrating forces. The excess of the former diminishes as the eccentricity diminishes; and when the orbit is reduced to a circle, the two sums become equal. By reason of such excess of resistance, hyperbolic and parabolic orbits will be reduced, first, to ellipses of great eccentricity; and second, by the same cause, the eccentricities and periodic times will continually diminish until the orbits become permanent and the sums of the two forces are equally balanced.

[If the foregoing propositions are presented as the basis of a theory of gravitation, it may be objected that the theory presents a phenomenon without a conceivable cause; viz., an elastic ether with a density *increasing* toward the centers of bodies immerced in it.

If gravitation is the result of ethereal pressure, the only conceivable mode of its manifestation is through a density *decreasing* toward the center.

Experiment proves that increased local vibration in an ether, or gas, is attended by diminished density at that point, and consequent motion toward the point of increased vibratory motion. Hence it would seem that the conclusion arrived at, that the density of the ether *decreases* as a function of the distance from the center of force increases, disproves the hypothesis.—Ed.]